Comparison of heating system alternatives for a rural house employing analytic hierarchy process

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Abstract. In this study were analyzed different heating systems to select the most appropriate type for a rural house. The performances of each alternative have been evaluated and compared using the analytic hierarchy process (AHP), a well-known and widely used multi-criteria method. There were considered five heating options: wood boiler, electric boiler, gas boiler, pellet boiler, and electric heating carpet, which were combined with two different heating terminals: radiant floor heating and the classic radiator. The alternatives were analyzed based on several criteria, which were defined considering economic, environmental, and social aspects. After the variants were ranked, and the most appropriate solution was determined, a sensitivity analysis was performed. The results indicated that the final ranking of the alternatives is robust, with slight variances, depending on the percentage with which the criteria weights were varied.

1. Introduction

There are different options that can be considered when choosing a heating system for a house and several factors that can influence the decision maker's opinion. The most important factor is to identify the heating sources that can be used to ensure thermal comfort. Considering the impact on the environment, the decision maker may choose between alternative heating sources (e.g., geothermal heat pumps, heating oil, solar heating) and traditional sources (e.g., wood, electricity, natural gas). Also, the heating sources may vary depending on the area where the house is located: if it is a mountain area or a rural zone wood is a reliable source, while electricity or natural gas is not that accessible; if the house is in a city, electricity or natural gas are the most obvious choices, because of the existing infrastructure, while wood may be considered expensive (because of the shipping and storage costs).

Different approaches were applied in previous related research papers to obtain the optimal solution for a heating system. In [1] the district heating planning tool (DHPT), developed considering the expected reduction in heat consumption, was used to evaluate five different district heating system development scenarios, to determine the most efficient variant (in Latvia). A similar study was conducted in [2], where six district heating systems from Upper Bavaria were monitored for a year and assessed by different Key Performance Indicators (KPIs) to identify the optimization potentials of each analyzed district heating system. In [3] were studied and evaluated different heating terminals (cast-iron radiator, radiant floor heating, bimetal radiator, and small temperature difference fan-coil unit) used in 'Coal-to-electricity' heat pump systems based on their heating performances. Also, the multi-criteria decision methods are frequently used to determine the most appropriate heating system for a given location. In [4] PROMETHEE (Preference Ranking Organization Method for Enrichment Evaluations) was used to evaluate four alternative energy heating systems for a single-family house in Kuujjuaq (Quebec, Canada), against twelve criteria, to identify the optimal solution. The Stochastic Multicriteria Acceptability Analysis (SMAA) was used in [5] to rank eleven heating systems alternatives for a sustainable residential area in Loviisa (Finland). The AHP (Analytic Hierarchy Process) and FCE (Fuzzy Comprehensive Evaluation) were used in [6] to evaluate the performances of five heating systems for a rural house located in North of China to select the best performing system.

In this paper, the AHP method was used by the authors to select the most appropriate heating system for a rural house, located in Satu-Mare county, Romania.

2. Methodology

AHP is a well-known multi-criteria decision-making method developed by T.L.Saaty to identify the optimal solution from a set of alternatives, based on multiple criteria analysis. The method involves several steps: the first two steps are to identify the problem, define the goal of the analysis, determine the criteria and the alternatives used to achieve the goal and construct the hierarchical representation of the problem [7,8]. The judgment matrix is constructed based on the decision maker's preferences, assessed by Saaty's major rating scale and contains the priority weights obtained after the pair-wise comparisons of the criteria [8,9]. Saaty's major rating scale measures the intensity of the importance of the criteria, from 1 (equal importance) to 9 (extreme importance) [7].

The judgment matrix A (pair-wise comparison matrix) is given below in (1), for n elements at each hierarchical level (criteria and alternatives), where a_{ij} represents a pair-wise comparison and is defined as a division between the weight value of criterion C_i and weight value of criterion C_j (i, j = 1, 2,..n) [8,10].

$$A = \begin{bmatrix} 1 & a_{12} & \cdots & a_{1n} \\ a_{21} & 1 & \cdots & a_{2n} \\ \vdots & \vdots & 1 & \vdots \\ a_{n1} & a_{n2} & \cdots & 1 \end{bmatrix} \tag{1}$$

The next step is to obtain the priority vector, which contains the relative weights used to form the ranking of importance for all decision elements, by normalizing the main eigenvector of judgment matrix A. This operation is repeated for each criterion and for all the alternatives to obtain the local and global priorities [10]. The advantage of the AHP method is that it allows to brief verification of the consistency of the judgments made by the decision maker. The pair-wise comparisons are considered consistent if the consistency ratio CR is less than 0.1. To calculate CR, the consistency index CI and the random index RI must be determined, using the largest eigenvalue of A (λ_{max}) [8,10,11]:

$$CI = \frac{\lambda_{\max} - n}{n-1} \tag{1}$$

$$RI = \frac{1.98 \cdot (n-2)}{n} \tag{2}$$

$$CR = \frac{CI}{RI} \tag{3}$$

The pair-wise comparisons matrix is considered consistent if $CR < 10\%$, otherwise the decision maker must revise the priorities established for each criterion and alternative, because the AHP method may not return the proper results. The final ranking matrix is obtained by synthesizing the local and global priorities for all the decision variables [10].

3. Case study

This case study was conducted based on the information obtained from the management of a company dealing with electrical installations work. The company has been contracted to install a new heating system for a house located in Satu-Mare county, Romania, having a constructed area of 120 square meters. Based on the preferences of the customer, nine alternatives were identified, considering five heating sources and two heating terminals : wood boiler combined with radiant floor heating (A1) or radiators (A2), electric boiler combined with radiant floor heating (A3) or radiators (A4), gas boiler

combined with radiant floor heating (A5) or radiators (A6), pellet boiler combined with radiant floor heating (A7) or radiators (A8) and electric heating carpet (A9).
The alternatives were evaluated against eleven criteria (C1-C11), which were defined considering economic, environmental and social aspects: raw material price ([RON]), boiler price ([RON]), installation price ([RON]), fuel price ([RON/year]), yield ([%]), boiler guaranty ([years]), system guaranty ([years]), system reliability ([years]), fuel availability ([%]), CO_2 emissions ([kg/year]), system autonomy ([points]).

4. Results

Satty's major rating scale was used to determine the judgment matrix, which was normalized to obtain the criteria weights. These first results are presented in table 1. The most important criterion is C4 (29.39 %), followed by C1 and C2 with the same percentage (15.93 %), and C3 with 11.29 %. Regarding the consistency of this judgment matrix (table 1), CR is less than 10% (3 %), which means the priorities established by the decision maker are consistent.

Table 1. Judgment matrix, criteria weights, and consistency results.

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	Criteria weights
C1	1	1	2	0.33	3	5	5	4	6	7	7	0.159
C2	1	1	2	0.33	3	5	5	4	6	7	7	0.159
C3	0.5	0.5	1	0.25	2	4	4	3	5	6	6	0.113
C4	3	3	4	1	5	7	7	6	8	9	9	0.294
C5	0.33	0.33	0.5	0.2	1	3	3	2	4	5	5	0.08
C6	0.2	0.2	0.25	0.14	0.33	1	1	0.5	2	3	3	0.38
C7	0.2	0.2	0.25	0.14	0.33	1	1	0.5	2	3	3	0.38
C8	0.25	0.25	0.33	0.17	0.5	2	2	1	3	4	4	0.056
C9	0.17	0.17	0.2	0.13	0.25	0.5	0.5	0.33	1	2	2	0.026
C10	0.14	0.14	0.17	0.11	0.2	0.33	0.33	0.25	0.5	1	1	0.018
C11	0.14	0.14	0.17	0.11	0.2	0.33	0.33	0.25	0.5	1	1	0.018
Consistency results				CI = 0.046				RI= 1.51			CR=0.0307	

The same procedure was used to determine all the comparison matrices, and their local weights of the alternatives analyzed. In tables, 2-4 are presented the judgment matrix, local weights and consistency results for the three most important criteria: fuel price (C4), raw material price (C1) and boiler price (C2).

Table 2. The comparison matrix, criteria weights, and consistent results concerning fuel price.

	A1	A2	A3	A4	A5	A6	A7	A8	A9	Local weights
A1	1	1	6	6	2	2	3	3	9	0.225
A2	1	1	6	6	2	2	3	3	9	0.225
A3	0.17	0.17	1	1	0.2	0.2	0.25	0.25	4	0.034
A4	0.17	0.17	1	1	0.2	0.2	0.25	0.25	4	0.034
A5	0.5	0.5	5	5	1	1	2	2	8	0.142
A6	0.5	0.5	5	5	1	1	2	2	8	0.142
A7	0.33	0.33	4	4	0.5	0.5	1	1	7	0.092
A8	0.33	0.33	4	4	0.5	0.5	2	2	7	0.092
A9	0.11	0.11	0.25	0.25	0.13	0.13	0.14	0.14	1	0.016
Consistency results				CI = 0.038			RI= 1.45			CR=0.026

According to the results presented in table 2, the most appropriate solution concerning fuel price is the wood boiler (A1 and A2), followed by the gas boiler (A5 and A6). The alternative A9 is positioned at the bottom of the ranking, which means it has the highest fuel price from all alternatives.

Table 3. The comparison matrix, criteria weights, and consistent results concerning raw material price.

	A1	A2	A3	A4	A5	A6	A7	A8	A9	Local weights
A1	1	0.5	1	0.5	1	0.5	1	0.5	0.25	0.061
A2	2	1	2	1	2	1	2	1	0.33	0.116
A3	1	0.5	1	0.5	1	0.5	1	0.5	0.25	0.061
A4	2	1	2	1	2	1	2	1	0.33	0.116
A5	1	0.5	1	0.5	1	0.5	1	0.5	0.25	0.061
A6	2	1	2	1	2	1	2	1	0.33	0.116
A7	1	0.5	1	0.5	1	0.5	1	0.5	0.25	0.061
A8	2	1	2	1	2	1	2	1	0.33	0.116
A9	4	3	4	3	4	3	4	3	1	0.293
Consistency results				CI = 0.004			RI= 1.45			CR=0.003

Table 4. The comparison matrix, criteria weights, and consistency results concerning boiler price.

	A1	A2	A3	A4	A5	A6	A7	A8	A9	Local weights
A1	1	1	1	1	1	1	3	3	0.33	0.107
A2	1	1	1	1	1	1	3	3	0.33	0.107
A3	1	1	1	1	1	1	3	3	0.33	0.107
A4	1	1	1	1	1	1	3	3	0.33	0.107
A5	1	1	1	1	1	1	3	3	0.33	0.107
A6	1	1	1	1	1	1	3	3	0.33	0.107
A7	0.33	0.33	0.33	0.33	0.33	0.33	1	1	0.2	0.038
A8	0.33	0.33	0.33	0.33	0.33	0.33	1	1	0.2	0.038
A9	3	3	3	3	3	3	5	5	1	0.284
Consistency results				CI = 0.006			RI= 1.45			CR=0.004

It can be noticed in tables 3 and 4 that the alternative A9 has the highest local weights concerning raw material and boiler prices, which means is the most preferred solution considering these two criteria.

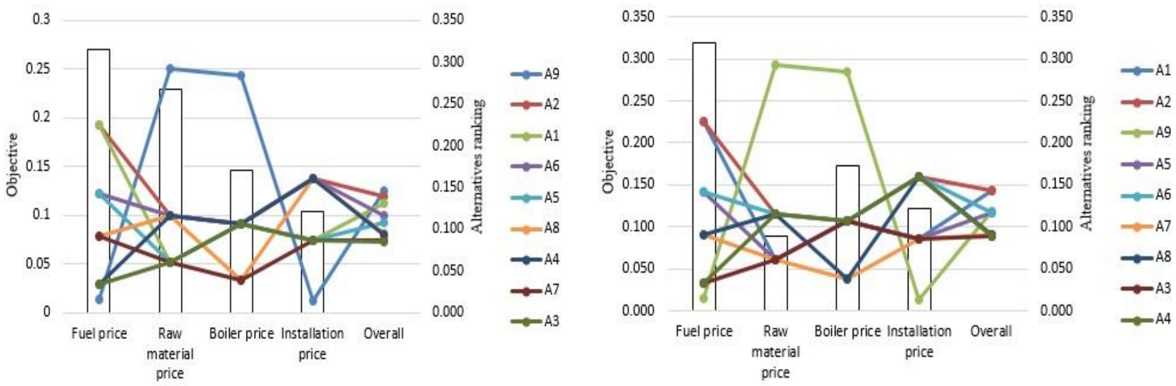
The local weights, obtained after performing the same procedure for the remained criteria, contributed to the construction of the final matrix, presented in table 5. The final ranking (Table 5), was obtained by multiplying the final matrix with the criteria weights and indicates the most appropriate heating system for a rural house, located in Satu-Mare county, Romania.

Table 5. Final results.

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	Final ranking
A1	0.061	0.107	0.086	0.225	0.058	0.112	0.204	0.176	0.168	0.209	0.029	0.138
A2	0.116	0.107	0.161	0.225	0.058	0.112	0.032	0.059	0.168	0.209	0.029	0.142
A3	0.061	0.107	0.086	0.034	0.185	0.058	0.204	0.176	0.096	0.025	0.163	0.087
A4	0.116	0.107	0.161	0.034	0.185	0.058	0.032	0.059	0.096	0.025	0.163	0.091
A5	0.061	0.107	0.086	0.142	0.106	0.112	0.204	0.176	0.021	0.05	0.163	0.113
A6	0.116	0.107	0.161	0.142	0.106	0.112	0.032	0.059	0.021	0.05	0.163	0.117
A7	0.061	0.038	0.086	0.092	0.058	0.112	0.204	0.176	0.168	0.209	0.063	0.088
A8	0.116	0.038	0.161	0.092	0.058	0.112	0.032	0.059	0.168	0.209	0.063	0.092
A9	0.293	0.284	0.014	0.016	0.185	0.211	0.053	0.059	0.096	0.018	0.163	0.132

It can be noticed that the optimal solution for the considered problem is A2, being the adequate alternative according to the analyzed criteria, followed closely by A1 and A9 with a tight score. Finally, the last places are occupied by A7 and A3 because they have the worst evolution.

A sensitivity analysis was performed to investigate the effect of varying the criteria weights of the four most important criteria, by seven percent. For the most part, the final ranking of the alternatives is robust, the alternatives A2, A1 and A9 change places between them if the criteria weights are varied only for the raw material price, results presented in figure 1. It can be noticed that the new ranking after the raw material price goes up is A9, A2, A1, A6, A5, A8, A4, A7, A3 (figure 1a), while the new ranking after the raw material price goes down is A1, A2, A9, A5, A6, A7, A8, A3, A4 (figure 1b).



a. b.

Figure 1. Sensitivity analysis with regards to raw material price change: a - upward, b - downward

5. Conclusions

The selection process of the most appropriate heating system is complex and requires the evaluation of various alternatives against multiple criteria, considering the beneficiary's preferences. This paper proposed an application of the AHP method to identify the optimal solution of a heating system for a rural house, located in Satu-Mare county, Romania. The results obtained suggest that the AHP may be an important tool to support the electrical designer, or even the beneficiary, in resolving this type of decisional problems. The results obtained, after the sensitivity analysis was performed, indicate that the ranking of the heating systems is robust and represents a reliable tool for the given purpose.

6. References

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